

## Graphs

### Introduction:-

Graph is a non linear data structure.  
For example:- Map.

A graph contains a set of points known as nodes (or vertices) and set of links known as edges (or Arcs) which connects the vertices.

### Graph Terminology:-

1. Vertex:- An individual data element of a graph is called as vertex.
2. Edge:- An edge is a connecting link between two vertices.

Edges are of three types:-

- 1) Undirected edge:- An undirected edge is a bidirectional edge.

2)

Unit :- 1

## Combinatorial Structure : Graph Theory Basics

1.1 :- Basic Terminology of Graph1.2 :- Simple Graph1.3 :- Degree of Vertex1.4 :- Degree Sequence of Graph1.5 :- First Fundamental Theorem of Graph1.6 :- Incident Matrix and Adjacent Matrix1.7 :- Trees : 1.7.1 Trees & their properties1.7.2 Binary Tree1.7.3 Complete Binary Tree1.7.4 Full Binary Tree1.7.5 Binary search TreeUnit :- 2Principals of Counting2.1 :- Principle of Inclusion and Exclusion2.2 :- Generalizing Inclusion - Exclusion Principles2.3 :- <sup>ran</sup> Dements - Nothing is in right place

## 2.4 :- Root Polynomials

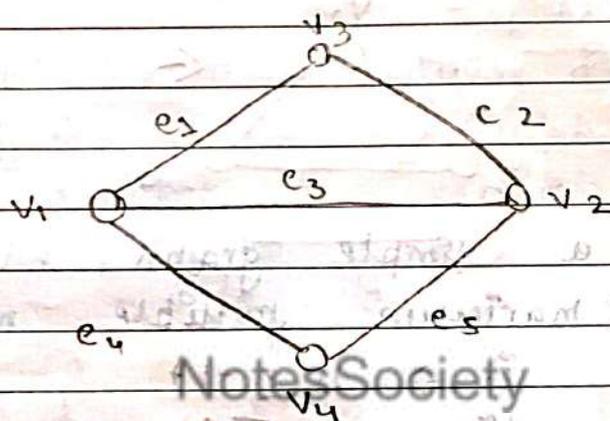
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## Unit Model:- Combinational Structures

1. Graph: The pictorial representation of object which are connected by link is known as graph.

→ Objects are known as vertices.

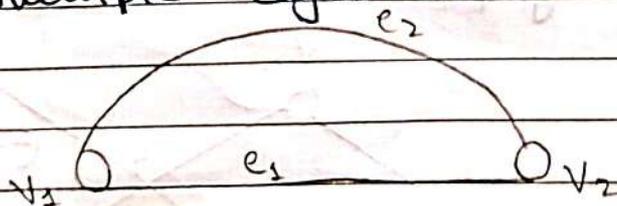
→ Links are known as edges



vertices =  $\{v_1, v_2, v_3, v_4\}$

Edges =  $\{e_1, e_2, e_3, e_4, e_5\}$

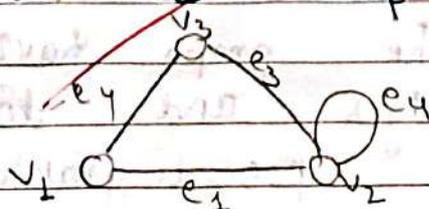
2. Parallel/Multiple edges: If between any pair of vertices, there are more than one edge. The edges are known as parallel/multiple edges.



Parallel edge

=  $\{e_1, e_2\}$

3. Loop: The edge starting from vertices, ending on the same vertices is known as loop.

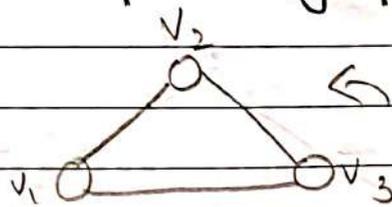


Loop =  $\{e_4\}$

#### 4) Types of Graphs:-

On the basis of parallel edge and loop:-

a) Simple <sup>graph</sup> graph:- The graph without any parallel edge and loop is known as simple graph.

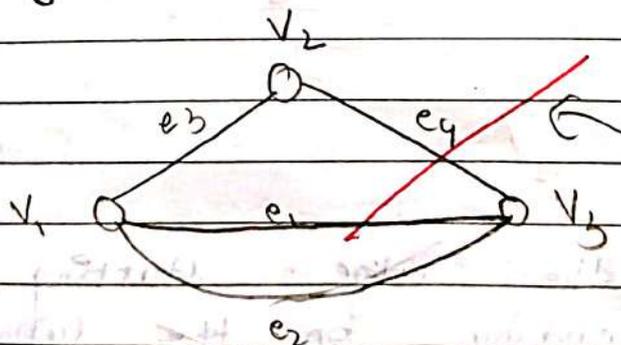


Simple graph.

Note:- In a simple graph, with  $n$  vertices the maximum possible number of edge is  $n(n-1)/2$ .

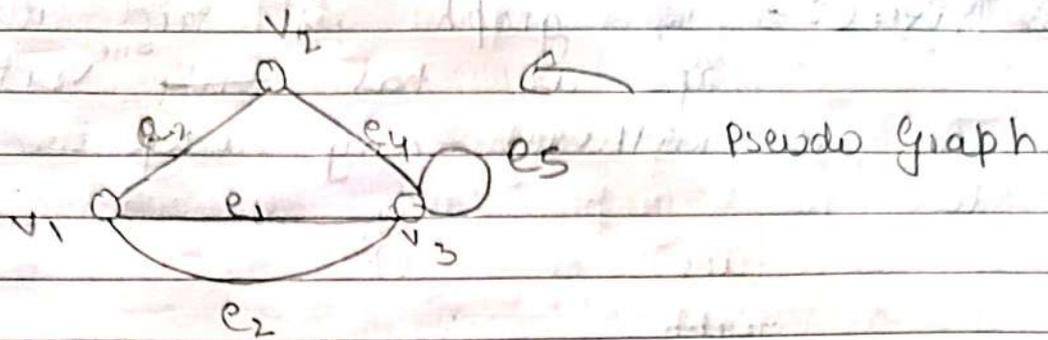
Eg:- The maximum possible number of edges :-  $n(n-1)/2 = 6/2 = 3$ .

b) Multiple graph:- The graph having multiple edges, is known as multiple graph.

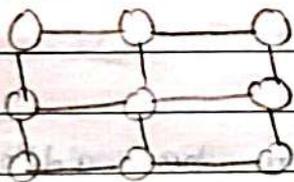


Multiple graph

c) Pseudograph:- The graph having multiple edges and loop is known as pseudo graph.

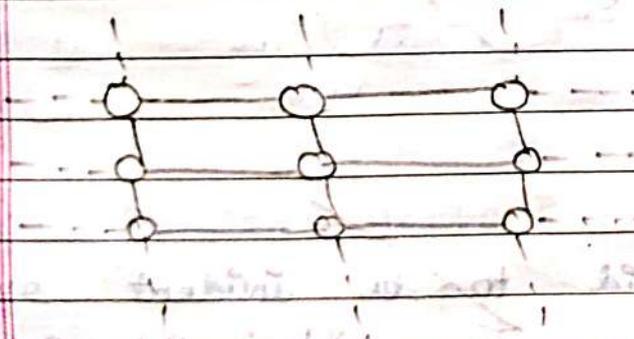


d) **Finite & Infinite graphs:-** A graph is having finite number of vertices and edges, is known as finite graph & a graph is having infinite number of vertices & edges, is known as infinite graph.



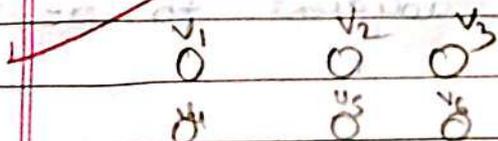
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finite graph



Infinite graph

e) **Null graph:-** A graph is said to be null, if it has any number of vertices without edges.



Null graph

f) Trivial? - A graph is said to be trivial, if it has <sup>one</sup> only vertex only, without any edge.

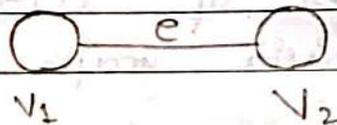


5) Adjacency:-

There are two types of adjacencies:-

a) Adjacent Vertices:-

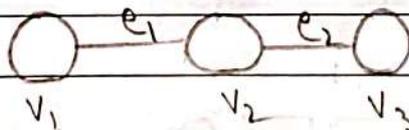
Two vertices are said to be adjacent, if they have one edge in common.



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b) Adjacent Edge:-

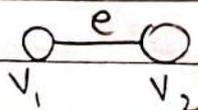
Two edges are said to be adjacent if they have one vertex in common.



6) Incidence:-

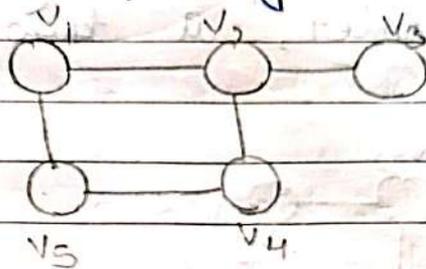
An edge is said to be incident on vertices ( $v_1$  &  $v_2$ ). If  $v_1$  &  $v_2$  are adjacent by  $e$ .

While neighbourhood of vertices  $v$  is set of all vertices which are adjacent to be  $v$ .



7) Degree of vertex:-

The degree of vertex  $v$  in a graph  $G(V, E)$  is the number of edges of edges incident on it.



$$dg(v_1) = 2$$

$$dg(v_2) = 3$$

$$dg(v_3) = 1$$

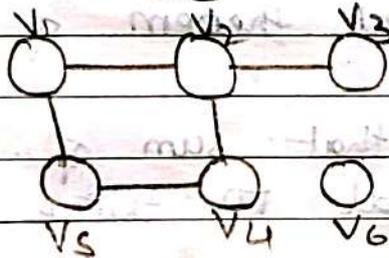
$$dg(v_4) = 2$$

$$dg(v_5) = 2$$

a) Pendant vertex:-

The vertex whose degree is one, is called pendant vertex.

Vertex  $v_3$  in fig is pendant vertex.



$$dg(v_3) = 1$$

b) Isolated vertex:-

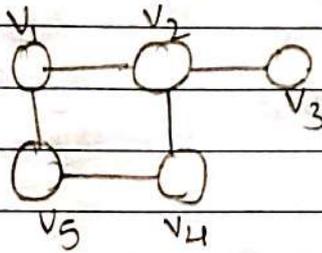
The vertex whose degree is zero, called isolated vertex.

Vertex  $v_6$  in fig is Isolated:- (above)

$$\therefore dg(v_6) = 0$$

8) Degree sequence of a graph:-

The set of degree of all the vertices of all the graph are arranged in increasing or decreasing order, is known as degree seq. of a graph.



Degree seq (increasing)  
1 2 2 2 3

OR  
Degree seq (decreasing)

3 2 2 2 1

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9) Hand shaking Theorem:-

(Fundamental theorem of graph theory).

a) If said graph that sum of degree of all vertices is equal to twice of number of its edges.

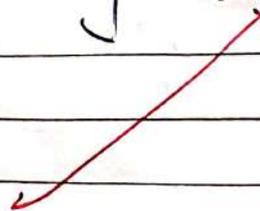
$$\sum \text{degree}(V) = 2m$$

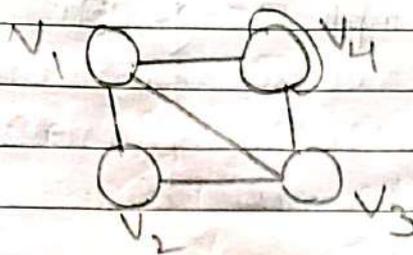
where m is no. of edges.

$$\sum 0 + 1 + 2 + 2 + 3 = 2m$$

$$\therefore m = 4$$

Verify the Handshaking theorem for a given graph.





$$dg(V_1) = 3 \quad dg(V_3) = 3$$

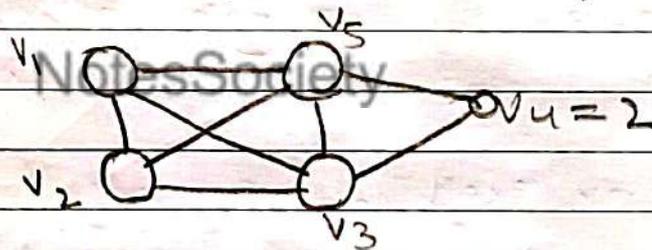
$$dg(V_2) = 2 \quad dg(V_4) = 4$$

$$\leq 3 + 2 + 3 + 4 = 2m$$

$$12 = 2m$$

$$6 = m$$

→ Verify the handshaking theorem for a given graph.



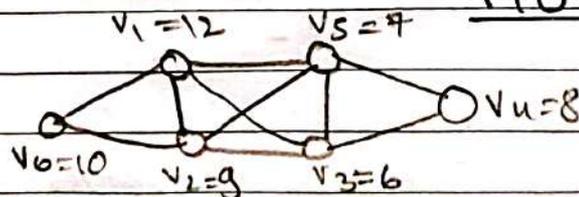
$$v_1 = 3, v_5 = 6, v_2 = 4, v_3 = 5$$

$$v_4 = 2$$

$$\leq 3 + 6 + 4 + 5 + 2 = 2m$$

$$20 = 2m$$

$$\boxed{10 = m}$$



$$v_1 = 12, v_2 = 9, v_3 = 6, v_4 = 8, v_5 = 7,$$

$$v_6 = 10$$

$$= \leq 12 + 9 + 6 + 8 + 7 + 10 = 2m$$

$$52 = 2m$$

$$26 = m$$

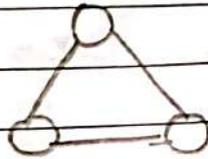
① Note :- In a simple graph, with  $n$  vertices, the maximum number of possible edges is :-  $n(n-1)/2$ .

Eg. 1  $n=3$

Find the maximum no. edges.

$$= \frac{3(3-1)}{2}$$

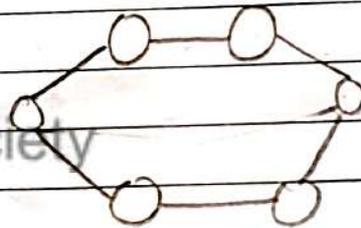
$$= 3$$



Eg. 2  $n=4$  (find max. p. edges).

$$\frac{4(4-1)}{2}$$

$$= 6$$



\*  $n=5$  (find max. p. edges)

② Note :- In simple graph with ' $n$ ' vertices, the maximum possible number of graph is constructed.

Formula :-  $\frac{n(n-1)}{2}$ .

Eg. 1 If  $n=3$ , then possible max no. of graph.

$$\Rightarrow \frac{3(3-1)}{2}$$

$$= \frac{3 \times 2}{2}$$

$$= 3$$

→  $n = 6$

$$\frac{6(6-1)}{2}$$

$$= 2^{15} = 32,768$$

③ Note:- The number of edges in a graph is known as its size, while the no. of vertices is known as its order.

### ⑩ Adjacent Matrix and Incident Matrix

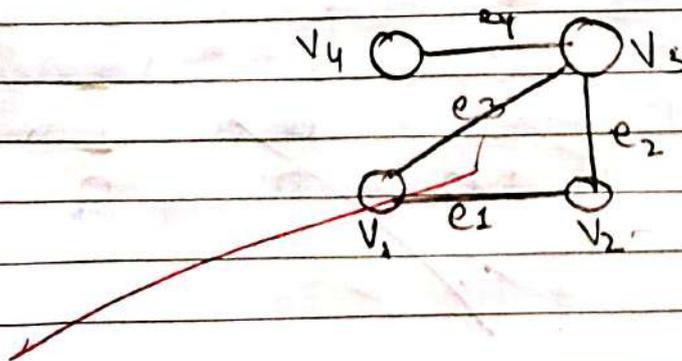
The matrix of the form,  $a_{ij} = \begin{cases} 1 & \text{if } v_i \text{ is adjacent to } v_j \\ 0 & \text{otherwise} \end{cases}$

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is known as adjacent matrix, while the matrix of the form,  $a_{ij} = \begin{cases} 1 & \text{if } e_i \text{ is incident to } v_j \\ 0 & \text{otherwise.} \end{cases}$

is known as incident matrix.

Find the matrix of incidence and adjacent



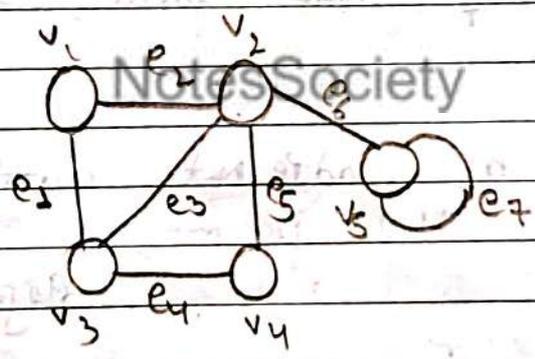
	$e_1$	$e_2$	$e_3$	$e_4$
$v_1$	1	0	1	0
$v_2$	1	1	0	0
$v_3$	0	1	1	1
$v_4$	0	0	0	1

Incidence

The matrix :-

$v_1$	0	1	1	0
$v_2$	1	0	1	0
$v_3$	1	1	0	1
$v_4$	0	0	1	0

adjacent

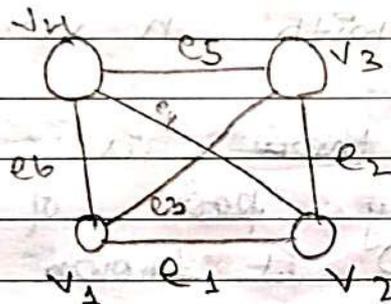


Incidence

	$e_1$	$e_2$	$e_3$	$e_4$	$e_5$	$e_6$	$e_7$	$e_8$
$v_1$	1	1	0	0	0	0	0	0
$v_2$	0	1	1	0	1	1	0	0
$v_3$	1	0	1	1	0	0	0	0
$v_4$	0	0	0	1	1	0	0	0
$v_5$	0	0	0	0	0	0	1	1

adjacent

	$v_1$	$v_2$	$v_3$	$v_4$	$v_5$
$v_1$	0	1	1	0	0
$v_2$	1	0	1	1	0
$v_3$	1	1	0	1	0
$v_4$	0	1	1	0	0
$v_5$	0	1	0	0	1



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Incidence:-

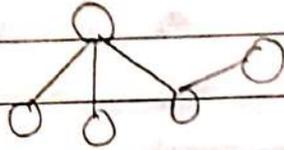
	$e_1$	$e_2$	$e_3$	$e_4$	$e_5$	$e_6$
$v_1$	1	0	1	0	0	1
$v_2$	1	1	0	1	0	0
$v_3$	0	1	1	0	1	0
$v_4$	0	0	0	1	1	1

Adjacency:-

	$v_1$	$v_2$	$v_3$	$v_4$
$v_1$	0	1	1	1
$v_2$	1	0	1	1
$v_3$	1	1	0	1
$v_4$	1	1	1	0

## Trees:

a) A graph without cycle, is known as a tree.

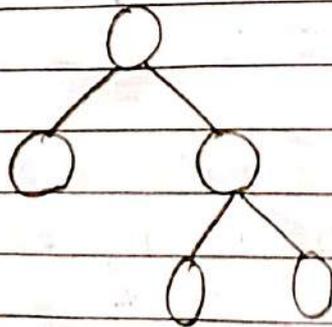


Tree.

b) Result:-

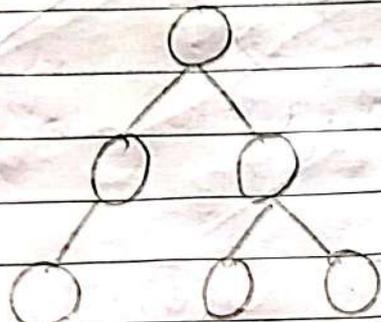
- In a tree with  $n$  vertices, there are  $(n-1)$  edges.
- In a tree there is only one path between every pair of vertices.
- The graph  $(G)$  is known as tree, if there is one and only one path between every pair of vertices.

c) Rooted Tree :- The tree is said to be a rooted tree, if we assign one of the vertices as a root and the rest are coming out from this.



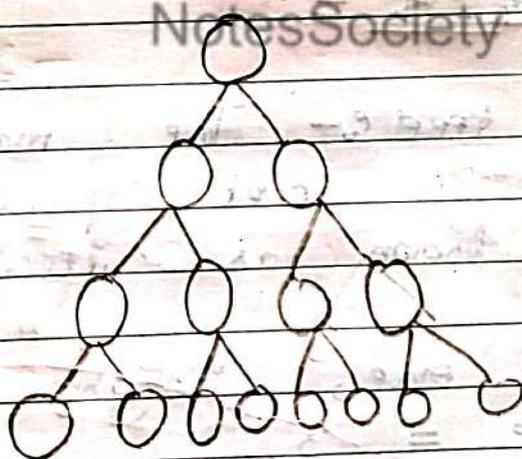
Rooted tree.

- Binary tree :- A rooted tree is said to be binary tree, if its out degree is less or equal to 2.  
i.e., ( $\leq 2$ ), i.e. 0, 1, 2.



Binary tree

- Complete binary tree :- A binary tree is said to be complete, if the out degree of every vertex is exactly equal to 2.



Complete binary tree.

- Level of a vertex :- The level of vertex in a rooted tree is its distance from root.

(PTO)

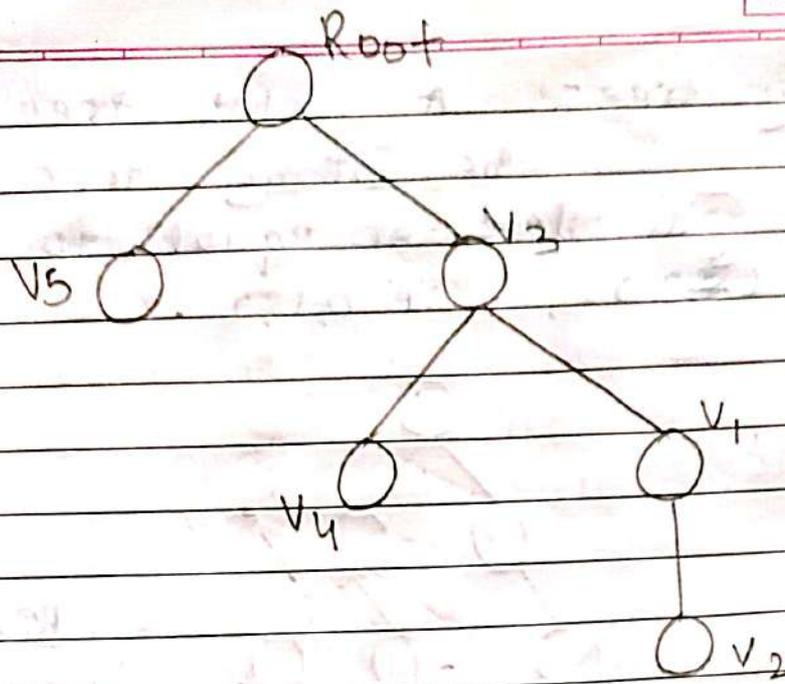


Figure 1:-

Level of  $V_1 = 2$

Level of  $V_2 = 3$

Level of  $V_3 = 1$

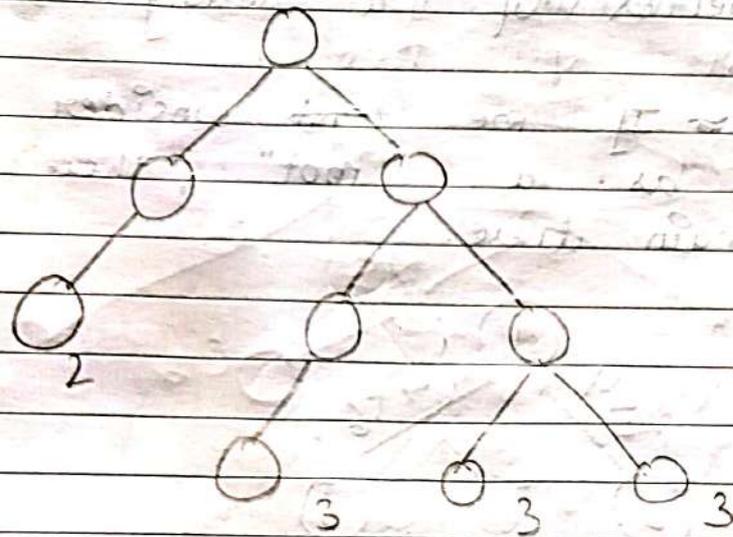
Level of  $V_4 = 2$

Level of  $V_5 = 1$

• Height of a tree :- The maximum of a rooted level of vertices in a tree is known as its height of a tree.

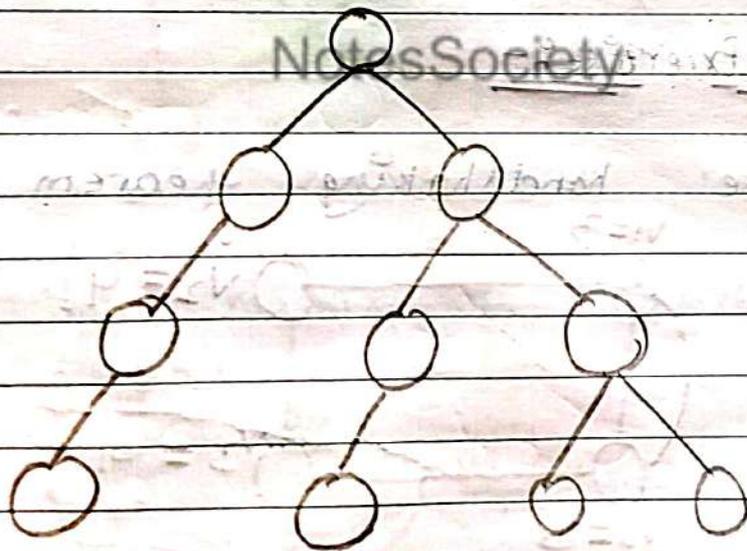
In figure one, height of a rooted tree = 3.

• Balanced tree :- A rooted tree is said to be a balanced tree if its height is "h" or  $(h-1)$ .



Balanced tree.

- Full binary tree :- A balanced tree is said to be full binary tree if its height is exactly "n"

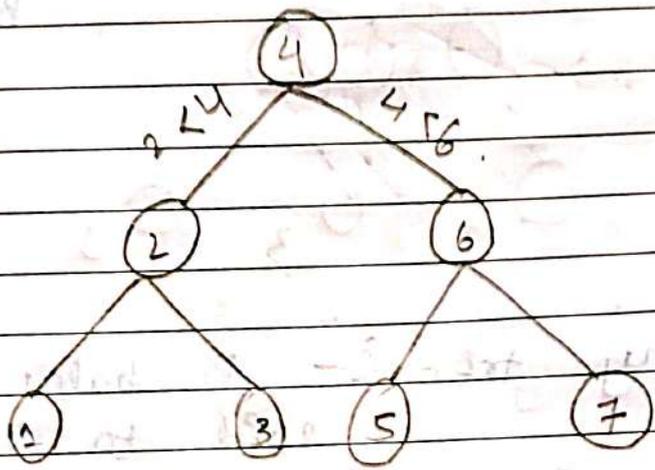


Full binary tree.

Binary Search Tree :- A binary search tree (BST) is a binary tree where every node in the left sub-tree is less than the root and every node in the right sub-tree is of a value greater than the root.

The properties of a binary search tree are

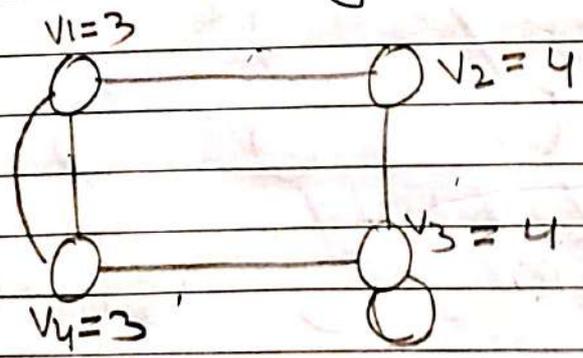
Recursive, If we ~~not~~ consider every node as a "root", these properties which remain true.



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Assignment / Exercise

Verify the handshaking theorem.



- dg.  $v_1 = 3$
- dg.  $v_2 = 4$
- dg.  $v_3 = 4$
- dg.  $v_4 = 3$

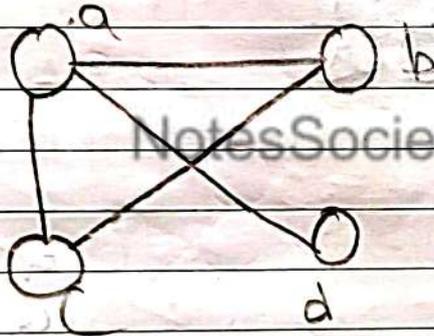
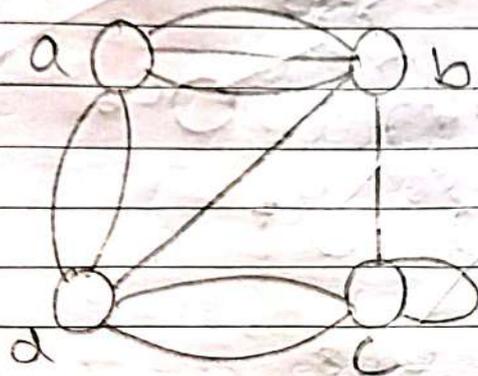
$$\sum v_i + v_2 + v_3 + v_4 = 2m$$

$$3 + 4 + 4 + 3 = 2m$$

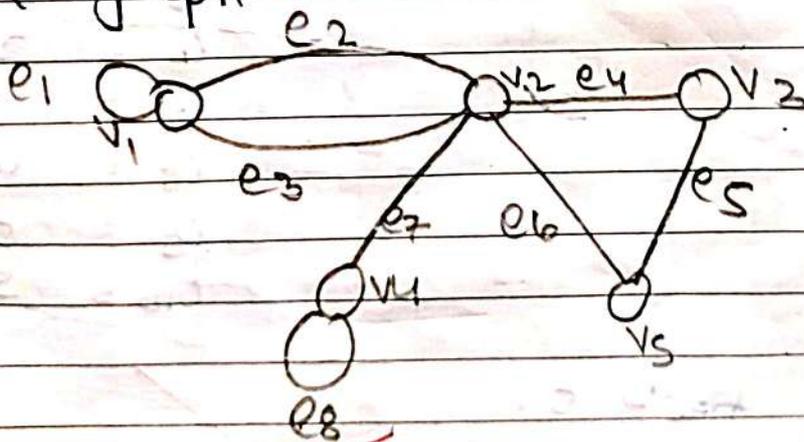
$$14 = 2m$$

$T = m$

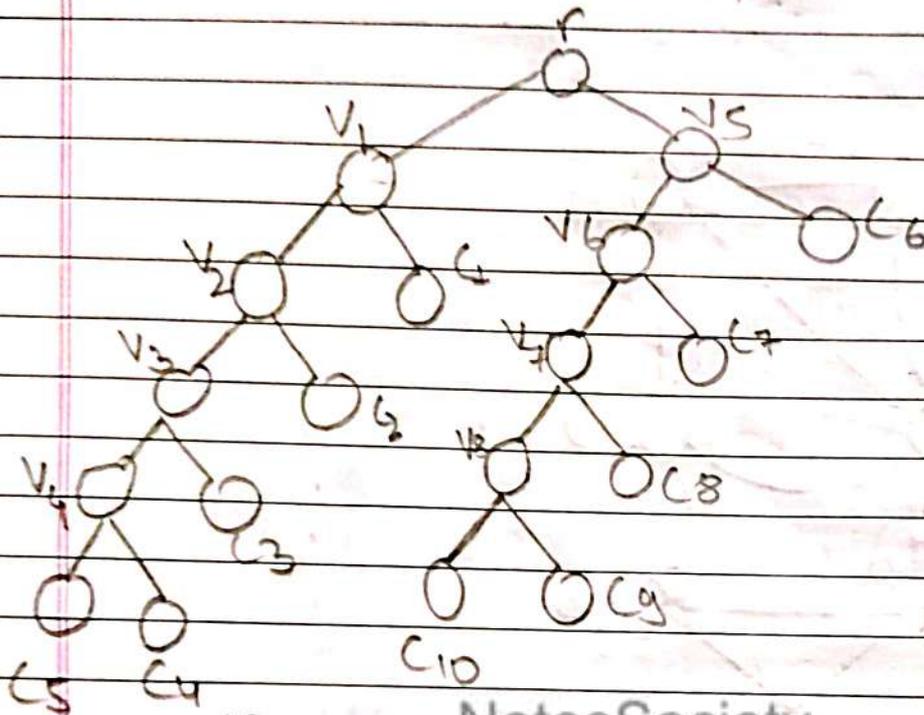
ii) Find the adjacent matrix for the following graph.



iii) Find the matrix of incidence for the given graph.



IV) Find the level of each vertex and the height of tree.



LEVELS. NotesSociety

$V_1 = 1$

$V_2 = 2$

$V_3 = 3$

$V_4 = 4$

$V_5 = 1$

$V_6 = 2$

$V_7 = 3$

$V_8 = 4$

$C_1 = 2$

$C_2 = 3$

$C_3 = 4$

$C_4 = 5$

$C_5 = 5$

$C_6 = 2$

$C_7 = 3$

$C_8 = 4$

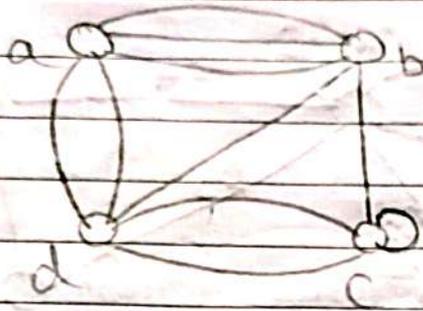
$C_9 = 5$

$C_{10} = 5$

Height of tree: 5.

2) adjacent matrix.

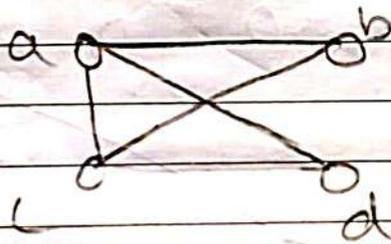
⇒



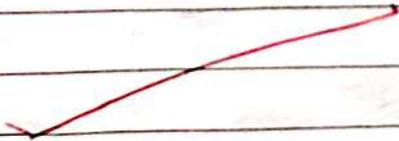
	a	b	c	d
a	0	1	0	1
b	1	0	1	1
c	0	1	1	1
d	1	1	1	0

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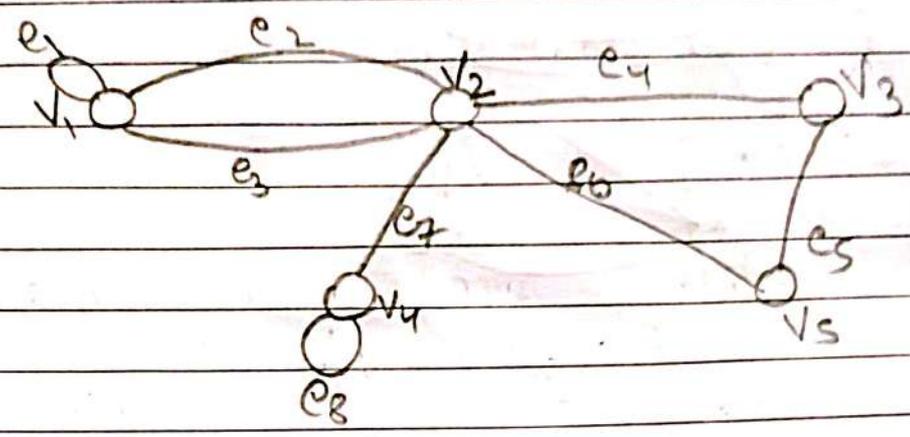
⇒



	a	b	c	d
a	0	1	1	1
b	1	0	1	0
c	1	1	0	0
d	1	0	0	0



3) Incidence matrix.



	$e_1$	$e_2$	$e_3$	$e_4$	$e_5$	$e_6$	$e_7$	$e_8$
$V_1$	1	1	1	0	0	0	0	0
$V_2$	0	1	1	1	0	1	1	0
$V_3$	0	0	0	1	1	0	0	0
$V_4$	0	0	0	0	0	0	1	1
$V_5$	0	0	0	0	1	1	0	0

~~15/10/24~~

## Module - II

### Principles of Countings

#### ① Set Theory

① Set theory - collection of well defined objects is known as set.

eg:  $A = \{x: x \text{ is a set of even numbers less than } 10\}$  (set builder form)

$B = \{2, 4, 6, 8\}$  (Roster form) (V)

$C = \{x: x \text{ is a set of criminal's}\}$  (x)

#### ② Types of sets

##### ① Finite and infinite sets:-

The set whose elements are finite is known as a finite set otherwise infinite set.

eg:  $F = \{x: x \text{ is a set of odd numbers less than or equal to } 1000\}$

$G = \{x: x \text{ is a set of real numbers}\}$

② Null set and universal set:- The set which doesn't contain any element is known as Null set. It is denoted as  $\{\emptyset\}$  or  $\{\}$  also known as  $\{\}$  (empty set).

If we have more than one set than the set containing all other sets known as universal set. It is denoted by  $U$ .

### (iii) Subset :-

Consider

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$A = \{4, 5, 6\}$$

$$B = \{6, 7, 8, 9\}$$

then A and B are said to be subsets of U because all the elements of A & B are containing elements of U.

### (iv) Operations on Sets :-

There are several operations for sets, some of them are :-

a) Union of sets

b) Intersection of sets

c) Difference of sets

eg:- let  $A = \{x : x \text{ is a set of natural numbers less than } k\}$

$B = \{x : x \text{ is a set of even numbers less than or equal to } 20\}$

$\therefore A \cup B$

In Roster form :-

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$$

$$B = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$$

$$\therefore A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 18, 20\}$$

$$A \cap B = \{2, 4, 6, 8, 10, 12\}$$

In set builder form:-

$A \cup B = \{x: x \text{ is a set of natural numbers less than or equal to } 14 \text{ and even numbers between } 16 \text{ to } 20\}$

$A \cap B = \{x: x \text{ is a set of even numbers less than or equal to } 12\}$

(v) Disjoint sets:- Two sets A and B are said to be disjoint sets if there ~~are~~ intersection is a Null set.

eg:-  $A = \{x: x \text{ is a set of odd numbers less than or equal to } 100\}$

$B = \{x: x \text{ is a set of even no. less than or equal to } 100\}$

$\therefore A \& B$  are disjoint sets.

$$\therefore A \cap B = \{\emptyset\}$$

(vi) Complement of a set:- The complement of set A is a set which contains all those elements which are not in A as compare to universal set.  
not in A as compared

eg:-  $U = \{a, b, c, d, e, f, g\}$

$$A = \{c, d\}$$

$$A^c = \{a, b, e, f, g\}$$

(vii) Cardinality of a set:- It is defined as no. of elements contained in a

$$A \cap B = \{2, 4, 6, 8, 10, 12\}$$

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⑦ Disjoint sets:- Two sets A and B are said to be Disjoint sets if their ~~are~~ intersection is a Null set.

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eg:-  $U = \{a, b, c, d, e, f, g\}$

$$A = \{c, d\}$$

$$A^c = \{a, b, e, f, g\}$$

⑨ Cardinality of a set:- It is defined as no. of elements contained in a

particular set. It is denoted as  $n(A)$  or  $|A|$ .

## ② Principle of Inclusion & Exclusion

The principle of inclusion & exclusion is used for joint sets  $P \in C$ , which are not disjoint sets.

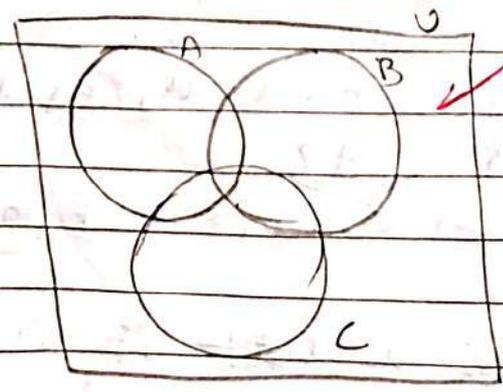
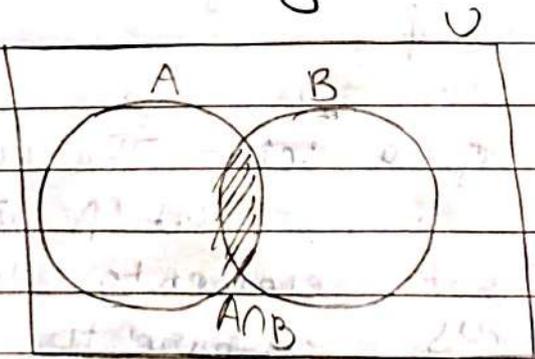
• For any two sets A & B :-

•  $|A \cup B| = |A| + |B| - |A \cap B|$

• For any three sets A, B & C :-

•  $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

This principle can be explained by using Venn diagram.



Example 1 :- An organization wants to hire the staff in which he needs 30 Technical and 20 non-Technical and 5 both which are Technical & Non-Technical. How many people are getting hired in total?

Sol<sup>n</sup> :- Let  $T = \text{Technical staff}$   
 $N = \text{Non-technical staff}$

$$n(T) = 30$$

$$n(N) = 20$$

$$n(T \cap N) = 5$$

∴ By Principle of inclusion & exclusion we have :-

$$n(T \cup N) = n(T) + n(N) - n(T \cap N)$$

$$n(T \cup N) = 30 + 20 - 5$$

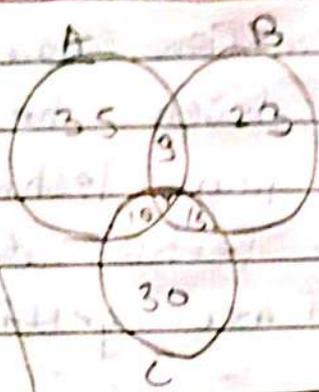
$$n(T \cup N) = 45$$

∴ Total staff hired = 45.

Example 2 :- There are 150 students in a party and there are three different brands A, B & C of drink out of which 58 students drink A, 49 drink B, 57 drink C, 14 drink A & C, 13 drink A & B, 17 drink B & C, 4 drink A, B & C. How many students drink none?

Method 1 -  ~~$U = 150$~~

$U = 150$



$150 - 124$

$= 26$

Method 2<sup>o</sup>

By the principle of inclusion exclusion, we have:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

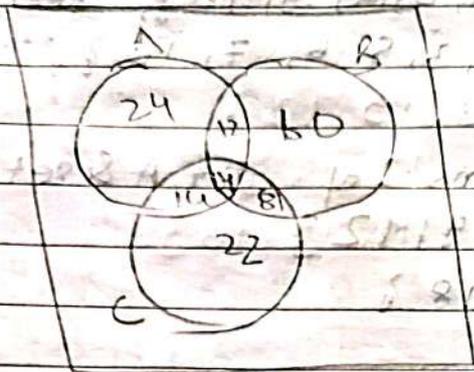
$$|A \cup B \cup C| = 164 - 40 = 124$$

$$\therefore |A \cup B \cup C| = U - |A \cup B \cup C| = 150 - 124 = 26$$

Example 3<sup>o</sup> - 270 college students, 60 like brussel sprouts, 94 like broccoli, 58 like cauliflower, 26 like both brussel sprouts & broccoli, 28 like both brussel sprouts & cauliflower, 22 like both broccoli & cauliflower, 14 like all three vegetables. How many of them don't like any of the three vegetables?

Q. 1

$$U = 270$$



$$= 270 - 154$$

$$= 116$$

Method:-2

By the principle of Inclusion & Exclusion, we have

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$= 24 + 60 + 22 - 14 - 11 - 8 + 4$$

$$= 154$$

$$\therefore |A \cup B \cup C| = U - |A \cup B \cup C|$$

$$= 270 - 154$$

$$= 116$$

Examples:-

1) If  $A = \{1, 4, 3, 7, 9, 12\}$ ,  $B = \{1, 8, 3, 4, 7\}$ . Find  $A \cap B$ .

$$\rightarrow A = \{1, 4, 3, 7, 9, 12\}$$

$$B = \{1, 8, 3, 4, 7\}$$

$$\Rightarrow A \cap B = \{1, 3, 4, 7\}$$

2. If set  $A = \{2, 4, 6, 8, 9, 12\}$ , set  $B = \{1, 3, 5, 9, 15, 18\}$  and set  $C = \{2, 3, 5, 6, 7, 10\}$ , solve following.

i) Find intersections of set A & set B.

$$\rightarrow A = \{2, 4, 6, 8, 9, 12\}$$
$$B = \{1, 3, 5, 9, 15, 18\}$$

$$\therefore A \cap B = \{9\}$$

ii) Find intersection of set B & set C.

$$\rightarrow B = \{1, 3, 5, 9, 15, 18\}$$
$$C = \{2, 3, 5, 6, 7, 10\}$$

$$\therefore B \cap C = \{3, 5\}$$

iii) Find intersection of set A and set C.

$$\rightarrow A = \{2, 4, 6, 8, 9, 12\}$$
$$C = \{2, 3, 5, 6, 7, 10\}$$

$$\therefore A \cap C = \{2, 6\}$$

3. If  $A = \{x, y, z\}$  and  $B = \{\phi\}$ . Find the intersection of two sets A and B.

$$\rightarrow A = \{x, y, z\}$$
$$B = \{\phi\}$$

$$\therefore A \cap B = \{\}$$

## Worksheet :-

1) State whether the following are True or False :-

a) If  $A = \{5, 6, 7\}$  and  $B = \{6, 8, 10, 12\}$ ;  $A \cup B = \{5, 6, 7, 8, 10, 12\}$ .

True.

b) If  $P = \{a, b, c\}$  and  $Q = \{b, c, d\}$ ; the intersection  $P \cap Q = \{b, c\}$ .

True.

c) Union of two sets is the set of elements which are common to both the sets.

False.

d) Two disjoint sets have at least one element in common.

False.

e) Two overlap sets have all the elements common.

False.

f) If two given sets have no elements common to both sets, the sets are said to be disjoint.

True.

g) If  $A$  and  $B$  are two disjoint sets then  $A \cap B = \{\}$ , the empty set. True.

2) Let A, B and C be three sets such that:-

$$\text{Set } A = \{2, 4, 6, 8, 10, 12\}$$

$$\text{set } B = \{3, 6, 9, 12, 15\}$$

$$\text{set } C = \{1, 4, 7, 10, 13, 16\}$$

Find:-

a)  $\{A \cup B\} =$  -

$$\{2, 3, 4, 6, 8, 9, 10, 12, 15\}$$

b)  $\{A \cap B\} =$  -

$$\{6, 12\}$$

c)  $\{B \cap A\} =$  -

$$\{6, 12\}$$

d)  $\{B \cup A\} =$  -

$$\{2, 3, 4, 6, 8, 9, 10, 12, 15\}$$

e)  $\{B \cup C\} =$  -

$$\{1, 3, 4, 6, 7, 9, 10, 12, 13, 15, 16\}$$

f) Is  $A \cup B = B \cup A$ ?

True.

g) Is  $B \cap C = B \cup C$ ?

False.

3) If  $A = \{1, 3, 7, 9, 10\}$ ,  $B = \{2, 5, 7, 8, 9, 10\}$ ,  
 $C = \{0, 1, 3, 10\}$ ,  $D = \{2, 4, 6, 8, 10\}$ ,  $E =$   
 $\{\text{negative natural numbers}\}$ , and  $F = \{0\}$ .

a)  $A \cup B = \{1, 2, 3, 5, 7, 8, 9, 10\}$

b)  $E \cup D = \{2, 4, 6, 8, 10\}$

c)  $C \cup F = \{0, 1, 3, 10\}$

d)  $C \cup D = \{0, 1, 2, 3, 4, 6, 8, 10\}$

e)  $B \cup F = \{0, 2, 5, 7, 8, 9, 10\}$

$$f) A \cap B = \{7, 9, 10\}$$

$$g) C \cap D = \{10\}$$

$$h) E \cap D = \{\emptyset\}$$

$$i) C \cap F = \{0\}$$

$$j) B \cap F = \{3\}$$

$$k) (A \cup B) \cup (A \cap B) = \{1, 2, 3, 5, 7, 8, 9, 10\}$$

$$l) (A \cup B) \cap (A \cap B) = \{7, 9, 10\}$$

4) IF  $A = \{2, 3, 4, 5\}$ ,  $B = \{c, d, e, f\}$ , and  $C = \{4, 5, 6, 7\}$ , Find:-

$$a) A \cup B = \{2, 3, 4, 5, c, d, e, f\}$$

$$b) A \cup C = \{2, 3, 4, 5, 6, 7\}$$

$$c) (A \cup B) \cap (A \cup C) = \{2, 3, 4, 5\}$$

$$e) \text{ Is } (A \cup B) \cap (A \cup C) = A \cup (B \cap C)?$$

$$\rightarrow A \cup (B \cap C) = \{2, 3, 4, 5\}$$

$$(A \cup B) \cap (A \cup C) = \{2, 3, 4, 5\}$$

$\therefore$  True.

5) IF  $A = \{a, b, c, d\}$ ,  $B = \{c, d, e, f\}$  and  $C = \{b, d, f, g\}$ , find:-

$$a) A \cap B = \{c, d\}$$

$$b) A \cap C = \{b, d\}$$

$$c) (A \cap B) \cup (A \cap C) = \{b, c, d\}$$

$$d) A \cap (B \cup C) = \{a, b, c, d\} \cap \{b, c, d, e, f, g\} = \{b, c, d\}$$

$$e) \text{ Is } (A \cap B) \cup (A \cap C) = A \cap (B \cup C)?$$

$\rightarrow$  True

~~12/11/24~~